a reprint from Applied Optics

Incoherent bandpass spatial filtering with longitudinal periodicity

Ting-Chung Poon and Guy Indebetouw

We explore the possibility of realizing bandpass filtering with longitudinal periodicity in incoherent systems. The necessary condition for spatial filtering to be longitudinally periodic is derived. Results indicate that bandpass filtering with longitudinal periodicity can be achieved in a two-pupil system with Fresnel zone plates with a small opening ratio as the pupils.

I. Introduction

The phenomenon of wave field replication without imaging elements is commonly known as the self-imaging effect. This effect has been observed and studied extensively[1–5] and found applications in areas such as interferometry,[6,7] spatial filtering,[8,9] and acoustooptics.[10] We explore the possibility of achieving spatial filtering with longitudinal periodicity in incoherent systems. By filtering with longitudinal periodicity we mean that a certain spatial filtering operation in the transverse directions (x,y) is repeated along the longitudinal direction (z) at a periodic distance. Such systems may find applications in 3-D information processing, coding, structured illumination, and in 3-D space variant filtering. Specifically, the aspect of incoherent bandpass filtering is emphasized in this paper. Optical transfer functions (OTFs) with bandpass characteristics cannot be synthesized in conventional single pupil systems. One, therefore, needs to use the method of pupil function replication[11] or employ two-pupil systems.[12–16] In Sec. II we develop the mathematical formalism used to describe the response of a defocused two-pupil system. The condition under which the OTF is longitudinally periodic is derived in Sec. III. Section IV describes how a Fresnel zone plate (FZP) can act as a self-imaging pupil. In Sec. V, we consolidate the results of Sec. III and IV to investigate bandpass filtering with longitudinal periodicity in a two-pupil system. Calculated results are obtained and shown to be consistent with theoretical predictions. Finally, in Sec. VI, are some concluding remarks.

Both authors are with Virginia Polytechnic Institute & State University, Blacksburg, Virginia 24061-0111; T. C. Poon is with the Bradley Department of Electrical Engineering, Image Processing Laboratory, and G. Indebetouw is with the Physics Department.

Received 4 December 1989.
0003-6935/90/233345-07$02.00/0.

II. Defocused OTF of Two-Pupil Systems

It is well known that the defocused OTF of an incoherent imaging system can be expressed as the autocorrelation of a defocused pupil function[17]:

$$\mathrm{OTF}(\bar{\rho},z) = P(\bar{\rho},z)\star P(\bar{\rho},z) = \int P(\bar{\rho}' - \bar{\rho})P^{*}(\bar{\rho}') \exp[j\pi\lambda z(|\bar{\rho}' - \bar{\rho}|^2 - \rho'^2)]d^2\rho'. \quad (1)$$

Here $P(\bar{\rho},z)$ is the defocused pupil function given by

$$P(\bar{\rho},z) = p(\bar{\rho}) \exp(j\pi\lambda z\rho^2), \quad (2)$$

where $p(\bar{\rho})$ is the in-focus pupil function, λ is the wavelength, and z is the defocused distance measured away from the focal plane of the second lens in Fig. 1.

For a given aperture function $A(\bar{r})$ located in the $\bar{r}$-plane as shown in Fig. 1, the corresponding defocused pupil, expressed in terms of the spatial frequency $\bar{\rho} = \bar{r}/\lambda f$, for a focal length f and in the paraxial approximation is given by

$$P(\bar{\rho},z) = A(\lambda f\bar{\rho}) \exp(j\pi\lambda z\rho^2). \quad (3)$$

For a two-pupil system[12,13] the defocused pupil is

$$P(\bar{\rho},z) = U(\bar{\rho},z) + V(\bar{\rho},z), \quad (4)$$

where $U(\bar{\rho},z) = u(\bar{\rho}) \exp(j\pi\lambda z\rho^2)$ and $V(\bar{\rho},z) = v(\bar{\rho}) \exp(j\pi\lambda z\rho^2)$. Thus the corresponding defocused OTF of the two-pupil system becomes, using Eq. (1),

$$\mathrm{OTF} = U\star U + V\star V + U\star V + V\star U. \quad (5)$$

Note that the autocorrelation terms are always of low pass characteristics. To achieve spatial filterings with properties other than low pass, the cross-terms need to be extracted. The cross-correlation of the interactive terms can be separated from the autocorrelation (noninteractive) terms by the use of a spatial frequency offset[18–20] or a temporal frequency offset.[21] A simple