

Capabilities and Limitations of Adaptive Canceling for Microwave Radiometry

Steven W. Ellingson, *The Ohio State University*, ellingson@ieee.org

Abstract— Radio frequency interference (RFI) limits the sensitivity of microwave radiometry by limiting the usable bandwidth and effective integration time of measurements. Adaptive canceling techniques offer a look-through capability that can potentially ease these constraints. In this paper, empirical arguments are used to determine the best possible attenuation of RFI for any canceler. Some field results using two different canceling methods are presented and shown to be consistent with this performance bound.

I. INTRODUCTION

Radio frequency interference (RFI) limits the sensitivity of airborne and space-based radiometers by constraining the usable bandwidth and effective integration time of measurements. Adaptive canceling techniques offer a look-through capability that can potentially ease these constraints. An adaptive canceler compares a reference signal $d(t)$ to the pre-detection sensor output $x(t)$, and uses the result to generate $\hat{z}(t)$, an estimate of the RFI waveform $z(t)$ in $x(t)$. Canceling is achieved by subtracting this estimate from $x(t)$. The various algorithms for adaptive canceling can be classified by (1) the method used to obtain $d(t)$, and (2) the method used to obtain $\hat{z}(t)$ from $x(t)$ and $d(t)$.

II. EMPIRICAL PERFORMANCE BOUNDS

Regardless of the particular adaptive canceling algorithm used, the degree to which RFI can be attenuated is limited by three factors: (1) INR_r , the interference-to-noise ratio in the “reference signal” $d(t)$; (2) INR_p , the INR in “primary signal” $x(t)$; and (3) the amount of data used to compute $\hat{z}(t)$. For the purposes of this study, the latter is most conveniently quantified as L , the number of data samples used under the condition that noise contribution is statistically independent between samples. Given these three parameters, one would like to know the maximum possible attenuation that can be achieved for a given interferer by subtraction of the best possible estimate of $z(t)$.

Consider a useful special case that provides insight into many other practical cases. Let $z(t)$ be $Ae^{j\omega t}$, where A is a complex constant. Also, assume for the moment that $d(t)$ is $z(t)$ plus ideal white gaussian noise (WGN). Let us further assume that we know we are dealing with a sinusoid, and have perfect *a priori* knowledge of ω . Then, $z(t)$ is completely described by a single unknown parameter, A . Assuming $\text{INR}_r \gg \text{INR}_p$, the optimum canceling

algorithm is as follows:

1. Estimate A from $d(t)$; call this \hat{A} .
2. Synthesize $\hat{z}(t) = \hat{A}e^{j\omega t}$.
3. Subtract $\hat{z}(t)$ from $x(t)$.

Due to our carefully chosen conditions, \hat{A} is very easy to compute. Given L samples of $d(t)$ at sample rate $1/T_S$, the answer is [1]:

$$\hat{A} = \frac{1}{L} \sum_{k=1}^L d(kT_S) e^{-j\omega kT_S} \quad (1)$$

Note that the performance of this algorithm is limited only by INR_r and L , not INR_p . Specifically, the mean attenuation, as determined in a Monte Carlo analysis, is equal to $\text{INR}_r L$.

A somewhat more realistic case is when $d(t)$ equals $H_{dz}z(t)$ plus noise, where H_{dz} is an unknown complex constant. Then, we must estimate H_{dz} as well as A to make the canceler work. The revised algorithm is:

1. Estimate the product $H_{dz}A$ from $d(t)$.
2. Estimate H_{dz} using the method described below.
3. Solve for A and synthesize $\hat{z}(t) = Ae^{j\omega t}$.
4. Subtract $\hat{z}(t)$ from $x(t)$.

In Step 2, H_{dz} is estimated as

$$\hat{H}_{dz} = \langle d(t)x^*(t) \rangle / \langle x(t)x^*(t) \rangle \quad (2)$$

where $\langle \cdot \rangle$ denotes the time-average of the argument and the superscript “*” denotes conjugation. To see why this is so, note that $\langle d(t)x^*(t) \rangle = H_{dz} \|z(t)\|^2$. Finally, note that we should expect that the canceling performance will now depend on INR_p as well as INR_r , since the noise in $x(t)$ degrades the estimate of H_{dz} when L is finite.

The mean attenuation achieved by this algorithm, as determined in a Monte Carlo analysis, is equal to the lesser of $\text{INR}_r L/4$ and $\text{INR}_p L$. The act of estimating H_{dz} has results in a 6 dB penalty in canceling performance, even when $\text{INR}_p = \infty$. This is because \hat{H}_{dz} depends on $d(t)$, which is affected by INR_r . Also, note that INR_p limits the performance that can be achieved with increasing INR_r .

In many practical situations, it may not be convenient or even possible to determine in advance what kind of interferer $z(t)$ actually is. Consider the following algorithm, which uses no *a priori* knowledge about $z(t)$ and assumes only that H_{dz} is a complex constant:

1. Estimate H_{dz} using Equation 2.

2. Synthesize $\hat{z}(t) = H_{dz}^{-1}d(t)$.
3. Subtract $\hat{z}(t)$ from $x(t)$.

It should be noted that Equation 2 is not the optimal estimator for all situations: We could possibly do better by taking into account the complete cross-spectrum of $x(t)$ with $d(t)$, as opposed to considering only the zero lag. Therefore, the observed performance will be a conservative estimate of the achievable performance.

A Monte Carlo analysis of the mean attenuation achieved by this algorithm reveals three interesting features. First, the penalty for not making use of *a priori* knowledge of $z(t)$ (or even the cross-spectrum of $d(t)$ with $x(t)$) is that the canceler stops working as the INR_r approaches and then drops below 0 dB. Second, we recover the 6 dB penalty on INR_r that we lost going from the optimum canceler to the second canceler considered above, as long as INR_r is sufficiently greater than 0 dB. Third, the “ $\text{INR}_p L$ ” bound remains in effect.

Taking into account the findings for each of the three algorithms considered above, we can conclude that the limit of attenuation is about $\min(\text{INR}_r L, \text{INR}_p L)$ as long as INR_r is sufficiently greater than 0 dB; the low- INR_r performance can be potentially much worse than this. This clearly demonstrates the value of *model knowledge* (that is, some *a priori* information that lets us parameterize $z(t)$ in a simple way) if good performance is desired.

Before continuing, it is worth noting a few practical issues that are most likely to degrade performance with respect to the ideal presented above. In short, these are: (1) noise correlation due to bandpass filtering, (2) non-stationarity of the signal statistics over the L samples used to compute $\hat{z}(t)$, and (3) differences in the response of the receivers or propagation channels associated with $x(t)$ and $d(t)$. Furthermore, recall that the above result was obtained using sinusoidal RFI waveforms, which are quite simple to manage. Other waveforms encountered in practice are more complex (e.g., modulated carriers, pulses) and so performance can be expected to be further degraded for these waveforms.

III. PERFORMANCE EXAMPLE

In this section, we consider the performance of adaptive canceling in field conditions. The data were obtained from the Rapid Prototype Array (RPA), a joint project of the University of California at Berkeley and the SETI Institute. The RPA is an array of 7 10-ft dishes located near Lafayette, CA. Each dish has independently-instrumented orthogonal linear feeds, and operates at L-band. The RFI in this experiment was from a GPS satellite. One antenna was pointed to put the satellite in the main beam. Data were obtained for both polarizations from this antenna using 8-bit A/Ds at 30 MSPS. The RPA’s receivers downconvert the nominal GPS center frequency of 1575.42 MHz to an intermediate frequency of 7.5 MHz before sampling. A

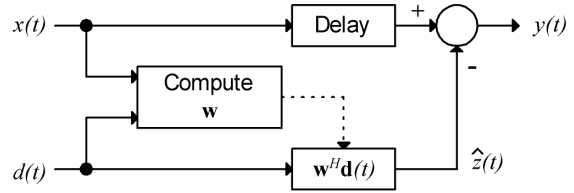


Fig. 1. MMSE canceler used in Section III.

filter with bandwidth of about 10 MHz was used for anti-aliasing. Additional information pertaining to the test conditions is available in [2].

Since the antenna points directly at the satellite, there is an INR improvement equal to the antenna gain, which is about 33 dB [3]. The strong C/A component of the GPS signal, which has a main lobe bandwidth of about 2 MHz, arrives with an INR of about -30 dB in its occupied bandwidth [4]. Also, since the GPS signal is circularly-polarized, it has approximately equal power in each receive polarization. Using one polarization as $x(t)$ and the other as $d(t)$, and taking into account the system temperature and polarization losses, $\text{INR}_p \approx \text{INR}_r \approx 3$ dB.

The adaptive canceler used in this experiment is shown in Figure 1. Here, $\hat{z}(t)$ is derived by filtering $d(t)$, which is represented in the figure as the operation $\mathbf{w}^H \mathbf{d}(t)$, where $\mathbf{d}(t)$ is an $M \times 1$ vector of samples of $d(t)$, and \mathbf{w} is a $M \times 1$ vector of filter coefficients which are obtained using the minimum mean square error (MMSE) algorithm [5]. In MMSE, $\mathbf{w} = \mathbf{R}^{-1} \mathbf{r}$, where $\mathbf{R} = \langle \mathbf{d}(t) \mathbf{d}^H(t) \rangle$ and $\mathbf{r} = \langle \mathbf{d}(t) x^*(t) \rangle$.

Figure 2 shows the results using $M = 8$ and $L = 1024$. No interferer-free data was available from which to estimate the bandpass shape (“baseline”), so the instead the output of the canceler was used to compute a fit. For spectroscopy, the result is quite impressive, with no sign of the RFI in the spectrum for integrations up to 3 ms. For total-power measurements, however, the canceler allows only about $300 \mu\text{s}$ of effective integration before the measurement again becomes RFI-limited. It is difficult to tell from Figure 2 alone whether the disappointing performance indicated by the bottom panel is due to error in the estimation of the spectral baseline or whether there might be some other factor involved. In a related study, however, the same performance was obtained for this scenario using a simulated version of the waveform, even when the baseline estimate was perfect. Using the simulated waveform with a perfectly flat bandpass was only slightly better (noise variance levels out at about 0.1 as opposed to 0.2), so the fact that the bandpass is not flat to begin with is not a major factor either. Thus, it is the canceler itself that is primarily limiting performance.

The attenuation of the interferer can be estimated from Figure 2 as about 15 dB (by comparing the top and middle panels) plus about 7 dB (from the bottom panel) for

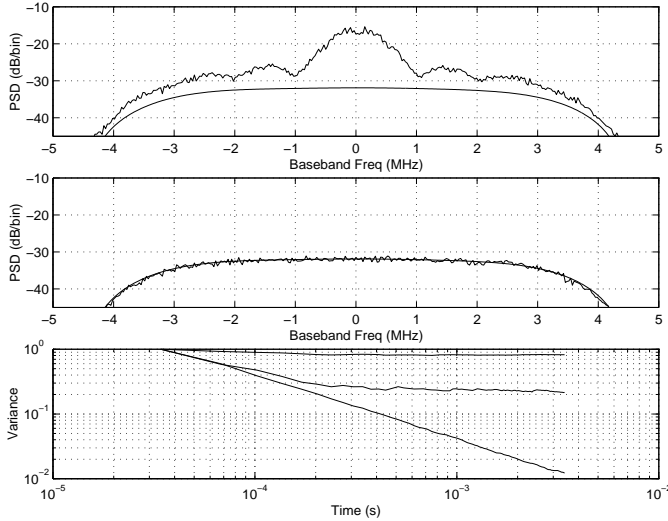


Fig. 2. MMSE Canceling. *Top*: Measured spectrum of $x(t)$ and estimated baseline. *Middle*: Same result, canceler on. *Bottom*: Noise variance for (*top to bottom*) canceler off, canceler on, and ideal WGN in the absence of RFI.

a total 22 dB of attenuation. The empirically-derived upper bound obtained in Section II applied to this case (with $L = 1024$ at 30 MSPS, and taking into account the band-limited noise spectrum) indicates that about 27 dB of canceling is possible. The reason for the shortfall is as follows: Although MMSE is optimum in the sense that it yields the maximum output signal-to-interference-plus-noise ratio (SINR), at low INR this is achieved by suppressing noise as well as RFI. Thus, the full effort of the MMSE algorithm is not applied to the task of attenuating RFI.

IV. AVOIDING THE USE OF A REFERENCE SIGNAL

It was noted in Section II that the performance of a canceler that does not use *a priori* information about the RFI waveform degrades quickly as INR_r is reduced below 0 dB. This would be the situation in the experiment above, for example, if the GPS satellite were in the sidelobes as opposed to the main beam. However, much *a priori* information is available about GPS: In fact, the GPS waveform is well-described by just a few slowly-varying parameters such as carrier phase, doppler shift, and code delay. Furthermore, $\hat{z}(t)$ can be obtained by computing the parameters directly from $x(t)$ (i.e., no $d(t)$ required), and then using the estimated parameters to synthesize a noise-free version of the waveform to subtract. This approach was successfully applied to the C/A component of GLONASS (which is very similar to C/A component of GPS) at another radio telescope [6].

When this approach is applied to the GPS data described above, the results are as shown in Figure 3. Although the amount of canceling is not significantly improved with respect to Figure 2, this has been achieved

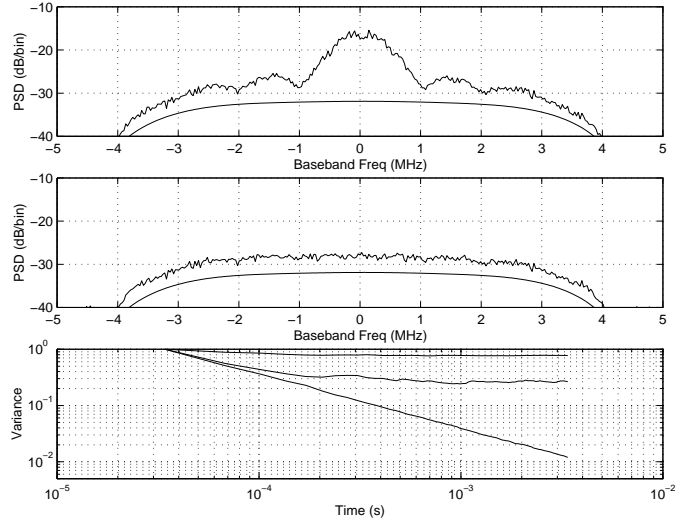


Fig. 3. C/A Canceling. *Top*: Measured spectrum of $x(t)$ and estimated baseline. *Middle*: Same result, canceler on. *Bottom*: Noise variance for (*top to bottom*) canceler off, canceler on, and ideal WGN in the absence of RFI.

without the use of a reference signal, and therefore INR_r is no longer an issue. The elevated noise baseline evident in the middle panel of Figure 2 is actually the “P” (wide-band) component of the GPS signal, which we have made no attempt to suppress. However, the P component has one-tenth the power spectral density of the C/A component and is nearly flat across the passband in this data, so it is still the INR_p -limited canceling of the C/A component that is limiting performance in this case. Techniques exist for removing the P component as well, if desired.

ACKNOWLEDGMENTS

Thanks to G. Bower of the University of California at Berkeley for providing the field data.

REFERENCES

- [1] S.M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*, Prentice-Hall, 1993.
- [2] G.C. Bower, “Application of Wiener and Adaptive Filters to GPS and Glonass Data from the Rapid Prototyping Array”, ATA Memo 31, August 2001.
- [3] G.R. Harp and R.F. Ackermann, “Mapping the Antenna Gain at the RPA”, ATA Memo 19, March 20, 2001.
- [4] E. Kaplan (ed.), *Understanding GPS: Principles and Applications*, Artech House, 1996.
- [5] S. Haykin, *Adaptive Filter Theory*, Prentice-Hall, 1986.
- [6] S.W. Ellingson, J. Bunton, and J.F. Bell, “Removal of the GLONASS C/A Signal from OH Spectral Line Observations Using a Parametric Modelling Technique”, *Astrophysical Journal Supplement*, Vol. 135, No. 1, pp. 87-93 (July 2001).