ECE 3254 PreLab 4 notes

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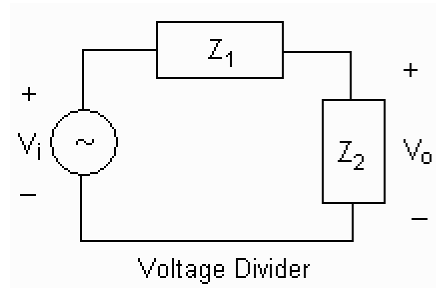
Voltage Divider



The basic voltage divider on the left will produce $V_o = V_i{*}Z_2 / (Z_1 + Z_2)$, where Z_1 and Z_2 may be complex.

Frequency Response

Capacitors (and inductors) change characteristic impedance as the frequency changes. In this lab, we will use a computer to capture the frequency response of RC filters by applying an AC frequency sweep to each filter input. During this sweep, the constant output voltage of a function generator is applied to the input of a circuit while the frequency is varied. The output voltage of the circuit is measured at each frequency step and the circuit response is plotted as Vo vs frequency, with a horizontal logarithmic frequency scale and either a linear voltage or a logarithmic dB (decibel) vertical scale.

The Decibel

dB is a logarithmic representation of gain or loss used to study or compare circuit responses. The most common form of dB uses a power ratio where $dB=10 \log (P_2/P_1)$. Circuit gains or losses are measured against a reference (P_1), usually the input power or voltage. Decibels are used extensively in electronic system calculations because the dB gains and losses of the response of antennas, mixers, filters, amplifiers, transmission paths, etc can be added together to obtain the total system gain or loss. The reference power may be any reference level – commonly used references are P_{in}, P_{max}, 1W (dBW), 1 mW (dBm), or 1V (dBV).

Using dB, ½ of the reference power (i.e. $P_{meas}/P_{ref} = ½$) would be 10log (½) = -3dB
some handy dB values that are frequently seen:

2x the reference power (a power gain of 2) would be 10 log (2) = 3 dB
4x the reference power = 6dB
10x the reference power = 10dB
100x the reference power = 20 dB.
½ the reference power = -3dB
¼ the reference power = -6dB
1/10 the reference power = -10dB
1/100 the reference power = -20dB

Most frequency response plots will use the dB scale, where gain in dB = $10 \log (P_2/P_1)$. We do not measure power in this lab, but $P = V_2R$. We can measure two voltages and if the same R is used for both voltage measurements, the R values will cancel and dB = $20 \log (V_2/V_1)$. The reference voltage V_1 can be input voltage, max output voltage, or some other value. When $V_2 / V_1 = 1/\sqrt{2}$, the output power of V_2 is ½ of the power of V_1 and 3 db below the power of V_1 (20 log ½ = -3).

Capacitor Impedance

The impedance of a capacitor is $Z_C = 1/j\omega C$. A capacitor's impedance will be $\infty\Omega$ when $\omega=0$, and the capacitor's impedance will be 0Ω when $\omega=\infty$. When capacitors are combined with resistors, the circuit's impedance and output voltage will be a function of frequency. There will be frequency (called the cutoff frequency) where $|P_o \text{ at cutoff}| = ½|P_o \max|$ or $|V_o \text{ at cutoff}| = \sqrt{½}\,|V_o \max|$ $(\sqrt{½} = 0.707)$.

Calculating The Cutoff Frequency Of A Circuit

Consider a simple low pass circuit where $V_o = V_i*X / (Y+j\omega Z)$.
V_o maximum $= V_i*(X / Y)$ when $j\omega Z = 0$. Note: X and Y must be real and not functions of ω.

Now, by definition the cutoff frequency occurs at half power (when $V_o = 0.707\, V_{max}$.) and we need to find where, in terms of ω, the magnitude of the output voltage produces this "half power".
So: $| V_o| = | V_i*X/(Y+j\omega Z)|$ and we want $| V_o|$ to equal $0.707\, V_{max} = 0.707\, V_i* (X/Y)$ at the cutoff freq.
Note: *this is NOT the same as* $|V_o|= 0.707$ *!!!*
Squaring the magnitude of both sides we get
$X^2/(Y^2+\omega^2Z^2) = (½)\,(X^2/Y^2)$ and the X terms cancel to give $1/(Y^2+\omega^2Z^2) = 1 /2Y^2$
Cross multiply and collect terms to get $Y^2 = \omega^2Z^2$ and now $Y = \omega Z$
where Y was the real part of the denominator and ωZ was the imaginary part of the denominator.
Solve for ω, then $f= \omega/2\pi$.

The high pass equation will have the form $V_o = V_i*X / (Y+1/j\omega Z)$ and the process is similar.

High Pass Filter Response

The high pass filter allows high frequencies to pass through to the circuit output but blocks lower frequencies.

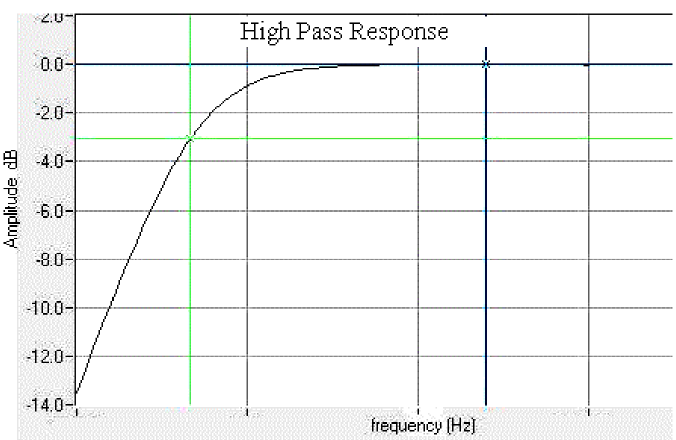


Figure 1

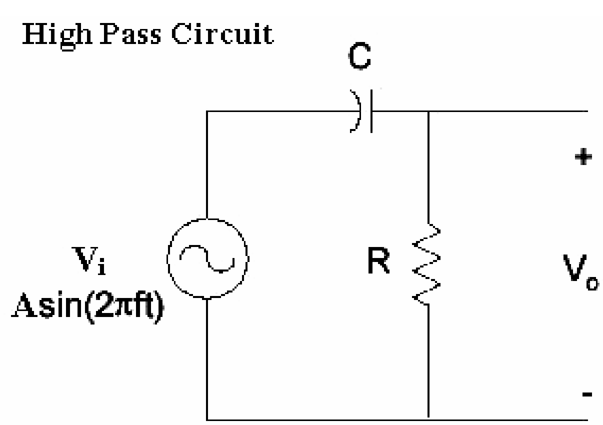


Figure 2

In the high pass response of fig 1 for circuit of fig 2, the cutoff frequency is marked by the green cursor where the amplitude is -3 dB. You can see that frequencies above the cutoff are passed because C is effectively a short circuit, and frequencies below the cutoff are increasingly attenuated as C becomes an open circuit. The response of this circuit is $V_o = V_i\, R/[\, R+1/j\omega C]$, and the maximum output voltage ($V_o = V_i$) occurs when $1/j\omega C=0$. The passband is defined as all frequencies above ω_{low}, and the passband gain is defined as the maximum filter amplitude in dB (0 dB in this case).

What happens if we add a resistor R_S in series with C?
C and the R_S become a complex impedance ($Z = R_S + 1/j\omega C$) in series with R.
By voltage division, $V_o = V_i\, R / (R + Z)$. With a little algebra the real part of the denominator becomes $R_S+ R$, the imaginary part becomes $1/j\omega C$, and we have $V_o = V_iR / [(R_S+ R) +1/j\omega C]$

The cutoff frequency will occur when the real part and imaginary part are equal.
In the passband Vo = ViR / [(R_S + R) because 1/jωC goes to 0 as ω becomes large.

Low Pass Filter Response

The low pass filter allows low frequencies to pass through to the circuit output but blocks higher frequencies.

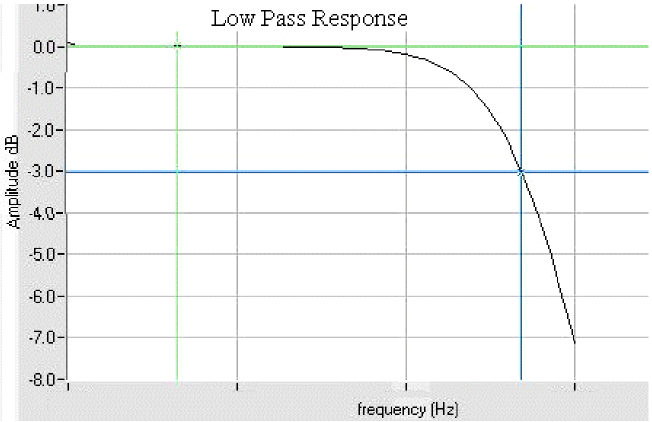


Figure 3

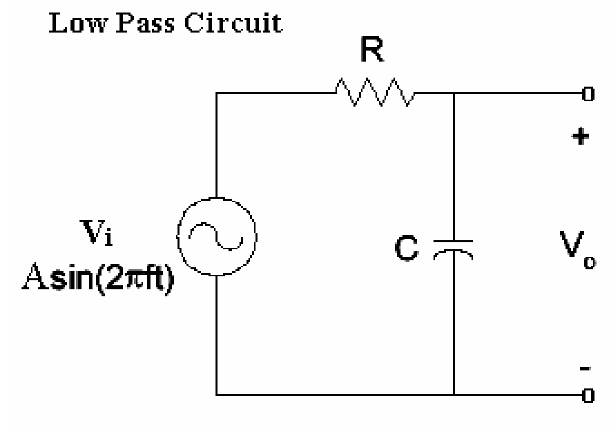


Figure 4

In the low pass response of fig 3 for circuit of fig 4, the cutoff frequency is marked by the blue cursor where the amplitude is -3 dB. You can see that frequencies below the cutoff are passed because C is effectively an open circuit, and frequencies above the cutoff are increasingly attenuated as C becomes a short circuit. The response of this circuit is Vo = Vi /(1+jωRC), and the cutoff frequency is ω_{high}= 1/ RC. The passband is defined as all frequencies below ω_{high}, and the passband gain is defined as the maximum filter amplitude in dB (0 dB in this case).

If we add a load resistor R_P in parallel with C, the parallel combination of R_P and C becomes Z = R_P/ 1+jωR_PC. By voltage division, Vo = ViZ / R + Z. With a little algebra the real part of the denominator becomes R_P + R, the imaginary part becomes jωR_PRC, and we have Vo = Vi R_P / [(R_P+R) + (jω R_PRC)].
The cutoff frequency will occur when the real part and imaginary part are equal. Hint R_PR / (R_P + R) = R_P || R.
In the passband Vo = ViR_P / [(R_P + R) because jωC goes to 0 as ω becomes very small.

Band Pass Filter Response

A band pass filter results from the combination of low pass and high pass circuits, and can be made tight enough to pass a narrow frequency range, such as a single radio station.

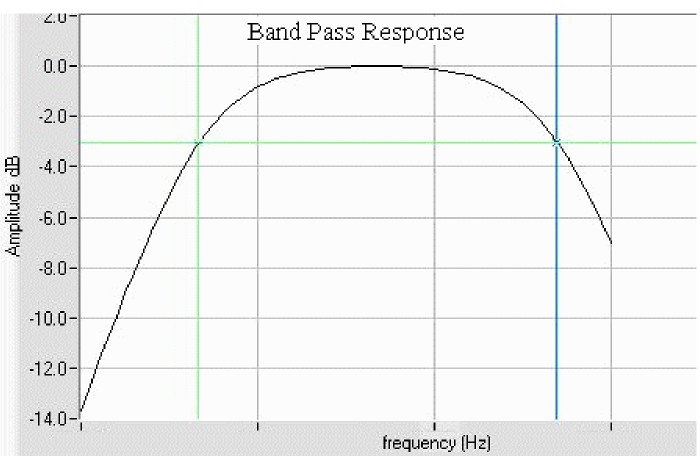


Figure 5

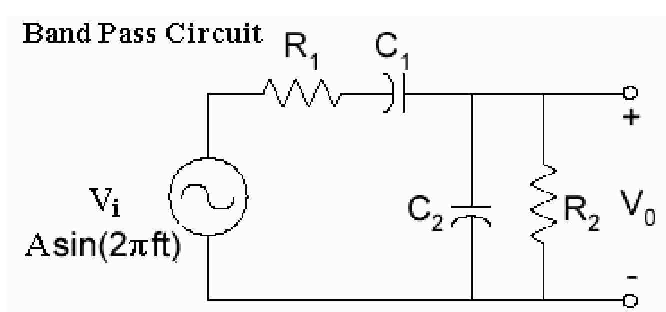


Figure 6

By combining the circuits of figure 2 and figure 4, we obtain the bandpass filter of figure 6.

For this bandpass filter, a highpass response (R_1, C_1, R_2) and a lowpass response (R_1, C_2, R_2) combine to produce $V_o = V_i R_2 / \{R_1 + R_2 + j\omega R_1R_2C_2 + 1/j\omega C_1\}$

If $C_1 >> C_2$, the output response shown in figure 5 has two cutoff frequencies $\omega_{low}= 1/ C_1(R_1+R_2)$ and $\omega_{high}= 1/ C_2(R1||R_2)$. In this circuit, both C_1 and C_2 are open at very low frequencies and there is no output voltage from the circuit. As frequency increases, C_1 begins to conduct and allow some of the input signal through the circuit. At frequencies above ω_{low}, C_1 is effectively a short circuit the filter produces its maximum output. As frequency increases to ω_{high}, C_2 begins to conduct, reducing the filter output voltage. As the frequency increases above ω_{high}, C_2 is effectively a short circuit and the filter output voltage steadily decreases (NOTE: C_1 is still a short circuit!). The passband is defined as the frequencies between ω_{low} and ω_{high}, and the passband gain is defined as the maximum filter amplitude in dB (0 dB in this case). The passband voltage gain $V_{oMax}/V_i = R_2 / \{R_1 + R_2\}$ because both capacitors are gone (C_2 open and C_1 short circuited) in the passband.

Measuring the Cutoff Frequency of a Circuit

The cutoff frequency of a circuit is defined as the frequency where output power is reduced to half of the maximum output power. For filters, the passband output voltage or power may be used as the reference (0 dB), placing the filter cutoff at -3dB. If the input voltage is used as the reference, the passband gain or loss = 20 Log Vo / Vi. The cutoff frequency will occur where the output drops to 3 dB below the maximum.

Because $P = V^2/R$, the equation for dB in terms of voltage becomes dB=20 log (V_2/V_1) when the resistor R is the same for both voltage measurements. The square relationship between power and voltage means that for ½ power, $V_{half\ power} = .707\ V_{reference.}$ Our lab measurements are voltages, so we will be using $dB = 10 \log (V_2^2/V_1^2) = 20 \log(V_2/V_1)$.

Your measurements may differ from the calculated cutoff frequencies due to resistor and capacitor tolerance, the stray capacitance in the protoboard and probes, and interpolation between captured data points with the cursors.

Modeling a filter in Pspice

Build the circuit in Pspice schematic (as you did for previous Labs) and save your file.

Draw > Get New Part > c > place, position, set the value, and wire the capacitor

Draw > Get New Part > VAC > place, position, set the value AC Mag = 3.54, and wire the AC source

Note: this sets the amplitude to 3.54 Vp, but, because we are not doing a transient analysis where the units are obvious and measurable, we will treat the voltages as RMS for simplicity.

Set up Analysis for AC sweep

Analysis > setup > uncheck all boxes, check AC Sweep >

Click AC Sweep button > set

AC Sweep Type = decade

Pts/Decade = 40

Start Freq = 1

End Freq = 10k

OK > Close > Run the sweep and debug if there are errors.

On the simulation results: Add a dB trace for Vout referenced to Vin..

[Trace > Add Trace (or click the add trace button)

> In Functions or Macros, select Analog Operators and Functions > DB()

> Under Simulation Output Variables, select V(C1:2), type /, then select V(R1:2) > OK]

*Note: Instead of using the drop downs, you may add the trace and manually enter the formula 20*LOG10(V(C1:2)/V(R1:2)) for the new trace*
Note: select the voltages that are correct for your schematic topology.

You should see three traces that look something like this:

When you have the dB trace completed, the cursor values placed, and your name on the simulation, print the page.

Adjust the components, wiring, and values for each filter type. For each version of the filter, make sure that you save your schematic.

Tip to save time:

- Add C2 to the first model, edit the information box at the bottom, and save the schematic as Prelab 4-9 (or something you can remember). Edit the simulation stop frequency and run the simulation of question #9 at this point. Save the schematic. Print the schematic and simulation.
- Then remove C1, replace C1 with a wire, and save the schematic as Prelab 4-7 (or something you can remember). Edit the simulation start and stop frequencies, and run the simulation for question #7. Save the schematic. Print the schematic and simulation.